Class-XII Miscellaneous Exercise. Matrix. 17-07-2020

P.1. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI+bA)^n = a^nI + na^{n-1}bA$. where I is the identity matrix of order 2 and $n \in N$.

Soln: It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

By principal of mathematical Induction,

Let $P(n): (aI+bA)^n = a^nI + na^{n-1}bA$

putting, $n=1$

$P(1): (aI+bA)^1 = a^1I + 1a^{1-1}bA$

$aI+bA = aI+bA$

$\Rightarrow$ Which is true for $n=1$

Let $n=K$,

$P(K): (aI+bA)^K = a^KI + Ka^{K-1}bA$ —— (1)

Let $n = K+1$

$P(K+1): (aI+bA)^{K+1} = a^{K+1}I + a^{K+1-1}_{(K+1)}bA$

Now, $L.H.S = (aI+bA)^{K+1} = (aI+bA)^K \cdot (aI+bA)$ $[\because a^{m+y} = a^m \times a^y]$

$= (a^KI + K \cdot a^{K-1}bA)(aI+bA)$

$= (a^KI + K \cdot a^{K-1}bA)(aI+bA)$

$[\because$ By (1) putting $(aI+bA)^K = a^KI + Ka^{K-1}bA)$

$= a^{K+1}I + Ka^KIbA + a^KIbA + Ka^{K-1}b^2A^2$

$= a^{K+1}I + Ka^Kb(IA) + a^Kb(IIA) + Ka^{K-1}b^2(A^2)$

$[\because AI = A$ and $A^2 = AA \Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0]$

$= a^{K+1}I + Ka^KbA + a^KbA + Ka^{K-1}b^2 \times 0 = a^{K+1}I + (K+1) \cdot a^KbA$

Therefore, the result is true for $n=K+1$ whenever it is true for $n=K$. So, by principle of mathematical induction, it is true for all $n \in N$.

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Class - XII Miscellaneous Exercise Matrix. 17-07-20

Q.2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, Prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, $n \in N$

Soln: It is given that $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

By principle of mathematical Induction.

Let $P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

Putting $n=1$

$P(1): A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix}$,

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ —— (I)

Let $n = K$

$P(K): A^K = \begin{bmatrix} 3^{K-1} & 3^{K-1} & 3^{K-1} \\ 3^{K-1} & 3^{K-1} & 3^{K-1} \\ 3^{K-1} & 3^{K-1} & 3^{K-1} \end{bmatrix}$ —— (II)

Putting, $n = K+1$, $P(K+1): A^{K+1} = \begin{bmatrix} 3^K & 3^K & 3^K \\ 3^K & 3^K & 3^K \\ 3^K & 3^K & 3^K \end{bmatrix}$

Now, $L.H.S = A^{K+1} = A^K . A^1$

$A^K . A^1 = \begin{bmatrix} 3^{K-1} & 3^{K-1} & 3^{K-1} \\ 3^{K-1} & 3^{K-1} & 3^{K-1} \\ 3^{K-1} & 3^{K-1} & 3^{K-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ [form (I) and (II)]

$= \begin{bmatrix} 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} \\ 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} \\ 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} & 3^{K-1}+3^{K-1}+3^{K-1} \end{bmatrix} = \begin{bmatrix} 3\times3^{K-1} & 3\times3^{K-1} & 3\times3^{K-1} \\ 3\times3^{K-1} & 3\times3^{K-1} & 3\times3^{K-1} \\ 3\times3^{K-1} & 3\times3^{K-1} & 3\times3^{K-1} \end{bmatrix}$

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Class-XII Miscellaneous exercise Matrix

Rest part of Q.2. 17.07.2020

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix} = RHS$$

Therefore, the result is true for $n=k+1$ whenever it is true for $n=k$. So, by principle of mathematical induction it is true for all $n \in N$.

Q.3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

Where n is any positive integer.

Soln: Let $P(n) = A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

Let $n=1$, $P(1) = A^1 = \begin{bmatrix} 1+2\times1 & -4\times1 \\ 1 & 1-2\times1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Which is true for $n=1$ —— (I)

Let $n=k$

$P(k) = A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ —— (II)

Let $n=k+1$

Then, $P(k+1) : A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k+4 \\ k+1 & -2k-1 \end{bmatrix}$

Now, $LHS = A^{k+1} = A^k A^1$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad [\text{by (I) and (II)}]$$

~~Let n=k+1,~~

~~Then, P(k+1) : A^{k+1} = [1+2(k+1) -4(k+1); k+1 1-2(k+1)] = [2k+3 -4k+4; k+1 -2k-1]~~

~~Now, LHS = A^{k+1} = A^k A'~~

~~= [1+2k -4k; k 1-2k] [3 -4; 1 -1]~~

$$= \begin{bmatrix} (1+2K)\cdot 3 + (-4K)\cdot 1 & (1+2K)(-4) + (-4K)(-1) \\ K\cdot 3 + (1-2K)\cdot 1 & K(-4) + (1-2K)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+2K & -4K-4K \\ K+1 & -1-2K \end{bmatrix} = \begin{bmatrix} 1+2(K+1) & -4(K+1) \\ K+1 & 1-2(K+1) \end{bmatrix}$$

Therefore, the result is true for $n = K+1$ whenever it is true for $n = K$. So, by principle of mathematical induction, it is true for all $n \in N$.

Q.4. If A and B are symmetric matrices, prove that $AB-BA$ is a skew-symmetric matrix.

Soln: It is given that A and B are symmetric matrices,

Then, $A' = A$ and $B' = B$

Now, $(AB-BA)' = (AB)' - (BA)'$ $[\because (A-B)' = A'-B'$ and $(AB)' = B'A']$

$= B'A' - A'B' = BA - AB$ $[\because B' = B$ and $A' = A]$

$= -(AB-BA)$

$\therefore (AB-BA)' = -(AB-BA)$

Thus, $(AB-BA)$ is a skew-symmetric matrix.

Q.5. Show that the matrix $B'AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

Soln: Let A is a symmetric matric, then $A' = A$

Let us consider $(B'AB)' = [B'(AB)]' = (AB)'(B')'$ $[\because (AB)' = B'A']$

$= B'A'(B)$ $[\because (B')' = B]$

$= B'(AB) = B'AB$ $[\because A' = A]$

$\therefore (B'AB)' = B'AB$ which shows that $B'AB$ is a symmetric matrix.

Now, Let A is a skew-symmetric matrix $[\because (AB)' = B'A'$ and $(A')' = A]$

Then $A' = -A$

Let $(B'AB)' = [B'(AB)]' = (AB)'(B')'$ $[\because A' = -A]$

$= (B'A')B = B'(-A)B = -B'AB$

$\therefore (B'AB)' = -B'AB$ which shows that $B'AB$ is a skew-symmetric matrix. Proved.